

# COIL SHAPE DIFFERENT FROM $\cos\theta$ SHAPE FOR OBTAINING PURE DIPOLE FIELD IN SUPERCONDUCTING MAGNETS

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## Abstract

It is well-known that a superconducting coil with a  $\cos\theta$  shape generates a pure dipole field. However, in practical magnets, one often comes across coil shapes quite different from  $\cos\theta$  shape. In an earlier work we optimized the coil shape of a dipole magnet which came out to be very different from the conventional  $\cos\theta$  shape. In this work we have found out the general analytical shape of a dipole coil.

## INTRODUCTION

Superconducting dipole magnets, producing very high field, are used in large numbers in high energy accelerators like LHC, RHIC, HERA etc. In these coil-dominated dipole magnets, the field quality depends on the shape of the superconducting coil. It is well-known that a long current-carrying conductor of circular or elliptic cross-section produces a perfect transverse field gradient inside [1]. This property is utilized for obtaining a pure dipole field by superposition of two opposite current densities. The coil shape is obtained by the displacement of two overlapping elliptic current distributions of opposite magnitudes. In such coils, the variation of coil width is similar to a  $\cos\theta$  distribution. Such magnets are called  $\cos\theta$  dipole magnets.

However,  $\cos\theta$  shape is not the only coil shape which can produce dipole field. One comes across dipole coil shapes which are markedly different from the  $\cos\theta$  shape [2]. In an earlier work [3], we designed coil shapes

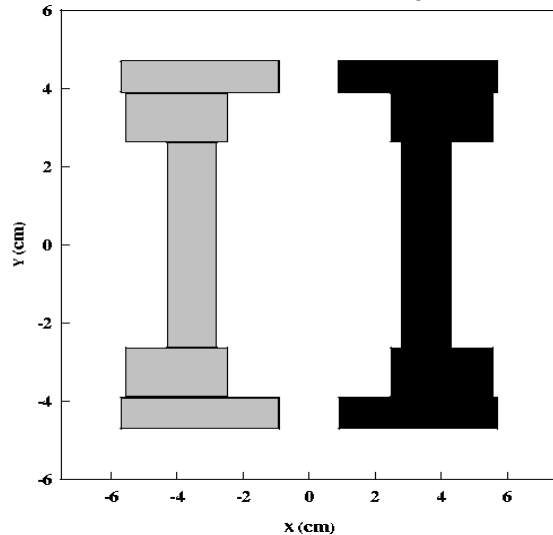


Figure 1: Optimized dipole magnet with block coils.

consisting of rectangular parts optimized for maximizing the field. As Fig.1 shows, the width of the coil in that case was smallest at the equatorial plane and increased away from that plane. This suggests the possibility of the existence of analytical coil shapes different from the  $\cos\theta$  shape. In this work we have worked out a general coil shape which can produce pure dipole field.

## FIELD INSIDE ELLIPTIC CONDUCTOR

The field  $B_y$  at a point  $(x_0, y_0)$  due to a current density  $J$  (along  $z$ ) in an infinitely long conductor is [4]

$$B_y(x_0, y_0, J) = \frac{\mu_0 J}{4\pi} \oint \ln[(x(y) - x_0)^2 + (y - y_0)^2] dy$$

where  $\mu_0$  is the permeability of air. Here  $x(y)$  defines the boundary of the coil as a function of  $y$ . The field and its components depend on the shape of the cross-section of the conductor. For an elliptic cross-section given by  $x = a \cos(\phi)$  and  $y = b \sin(\phi)$ , the field inside the conductor is given by

$$B_y(x_0, y_0, J) = \mu_0 J \frac{b}{a+b} x_0 \quad (1)$$

This is a field with a constant gradient. A pure dipole field is obtained by superposing two displaced current distributions with a displacement of  $t$  for each.

$$\begin{aligned} B_y(x_0, y_0) &= B_y(x_0 + t, y_0, J) - B_y(x_0 - t, y_0, -J) \\ &= 2\mu_0 J t \frac{b}{a+b} \end{aligned}$$

## GENERAL COIL SHAPE FOR PRODUCING PERFECT DIPOLE FIELD

Here we investigate if we can obtain pure dipole field using any other cross-section for the conductor. Let us investigate the following conductor shape for this purpose.

$$x = a \cos(\phi) + c \cos(2\phi) + d \cos(3\phi)$$

and

$$y = b \sin(\phi) - c \sin(2\phi) - d \sin(3\phi)$$

where the parameters  $c$  and  $d$  cause deviation from the elliptic shape. With this,  $B_y$  comes out to be

$$B_y(x_0, y_0, a, b, c, d, J) = \frac{\mu_0 J}{4\pi} \int_{-\pi}^{\pi} \ln \left[ (x(\phi) - x_0)^2 + (y(\phi) - y_0)^2 \right] \frac{dy(\phi)}{d\phi} d\phi$$

$$= \mu_0 J \left[ \frac{-\frac{c(a-b+3d)}{a+b} + \frac{b}{a+b}x_0 + \frac{3d(a-b)}{(a+b)^2}x_0 + \frac{2c}{(a+b)^2}(x_0^2 - y_0^2) - \frac{4d}{(a+b)^3}(x_0^3 - 3x_0y_0^2)} \right]$$

This is a combination of dipole, quadrupole and sextupole fields. One can easily remove the quadrupole field component by mixing two fields from two different current distributions as shown below.

$$B_y(x_0 + t, y_0, a, b, -c, d, J) + B_y(x_0 - t, y_0, a, b, c, d, -J) = 2\mu_0 J \left[ \frac{-\frac{c(a-b+3d)}{a+b} + \frac{bt}{a+b} + \frac{3d(a-b)t}{(a+b)^2} - \frac{2ct^2}{(a+b)^2}}{\frac{4dt^3}{(a+b)^3} - \frac{2}{(a+b)^2} \left( c + \frac{6dt}{a+b} \right) (x_0^2 - y_0^2)} \right]$$

This field is a combination of a dipole field and a sextupole field ( $\propto x_0^2 - y_0^2$ ). The sextupole term vanishes when we take  $c = -6dt/(a+b)$ . With this condition, the dipole field comes out to be

$$B_y(x_0, y_0) = 2\mu_0 J t \left[ \frac{b}{a+b} - \frac{3d(a-b+6d)}{(a+b)^2} + \frac{8dt^2}{(a+b)^3} \right]$$

As evidenced from the absence of  $x_0$  and  $y_0$  in the equation, the field is constant and therefore completely uniform within the overlapped area.

Fig.1 shows typical coil shapes for various values of  $a$ ,  $b$ ,  $d$  and  $t$ . Fig.2 shows the field line plot calculated with SUPERFISH code for a dipole with  $a=6.4$ ,  $b=6.0$ ,  $d=-1$  and  $t=1.2$ .

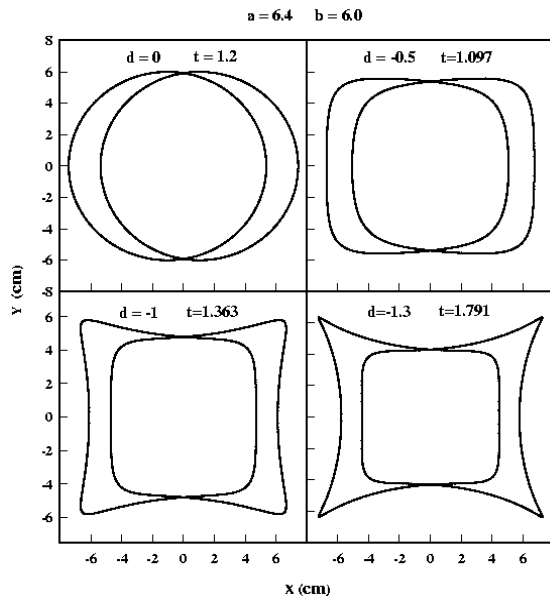


Figure 2: Various coil shapes for pure dipole field.

## DISCUSSIONS

Since the field amplitude depends non-linearly on the shape parameters  $a$ ,  $b$ ,  $d$ , and  $t$ , one can try to maximize the field for a given coil area and current density. This is equivalent to minimizing the coil area for a required magnetic field.

A FORTRAN code, based on random search optimization, has been written for this purpose. For a field of 7.5 T in an aperture of 5cm radius, with an engineering current density of 750 A/mm<sup>2</sup>, the coil area is 21.54 cm<sup>2</sup>. For  $\cos\theta$  shape, it is 22.86 cm<sup>2</sup>.

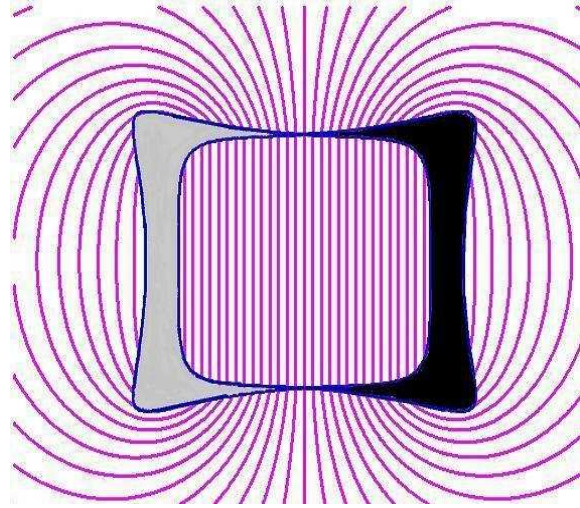


Figure 3: Pure dipole field inside with new coil.

We have shown that the coil shape obtained by the overlapping of two displaced ellipses is not the only shape which can produce a pure dipole field. The shape given in this work also produces a perfect dipole field. Depending on the values of the shape parameters, the coil shape can be markedly different from the  $\cos\theta$  shape.

Moreover, the parameters involved in the shape equations can be optimized for maximizing the field or minimizing the coil area. The coil area can be reduced by about 7% for a field level of 10 T with the new shape as compared to the conventional shape. This will mean a reduction in the quantity of the costly superconducting wires. But, of course, reduction in coil area is not the only criterion in superconducting magnets. One has to see that the field at all the points in the coil is below the acceptable critical field. We have not delved into this aspect in this work.

## REFERENCES

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