ESTIMATION OF EMITTANCE GROWTH IN 10 MEV HIGH CURRENT COMPACT CYCLOTRON

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Abstract

A preliminary study on estimation of emittance growth during the injection in a 10 MeV compact cyclotron is described by assuming cyclotron as a smooth focusing channel. The axial emittance growth has been studied as a function of beam current, injection energy and injected beam radius to obtain the optimized parameters for minimum emittance growth.

INTRODUCTION

Emittance growth of the space charge dominated beam is one of the most important topics in high intensity accelerator because it causes severe beam loss during transport. At VECC, we are developing a 2.45 GHz microwave ion source, which will deliver 20mA of protons at 80keV. This beam will be transported by a low energy beam transport line and will be injected axially into a 10 MeV, 5-10mA compact cyclotron [1].

Several theoretical workers [2, 3] have studied the emittance growth in a linear focusing channel for non-uniform and mismatched beam. Results of studies show that the non-uniform beam has higher field energy than the equivalent uniform beam. This difference in field energy converts into transverse kinetic energy and is responsible for the emittance growth. In this work we first explain the analytical formulations used to calculate the emittance growth for a non-stationary beam during the injection in a high current compact cyclotron. We then estimate the emittance growth for various machines and beam related parameters and minimize the emittance growth by optimizing the parameter of the compact cyclotron.

THEORETICAL ANALYSIS

As mentioned earlier, in the analysis we have assumed cyclotron as a smooth focusing channel. The stationary state of the beam distribution in such a system is characterized by uniform density and constant rms radius a_i satisfying the condition

$$k_i^2 a_i - \frac{\varepsilon_n^2}{\beta^2\gamma^2 a_i^3} = 0 \ , \quad k_i^2 = k_0^2 - \frac{K}{a_i^2} \qquad (1)$$

Here $K = 2I\left(2\pi/\Delta\phi\right)/I_0\left(\beta\gamma\right)^3$ is the generalized perveance, I is the beam current, $\Delta\phi$ is the phase acceptance, k_i is the average focussing strength with space charge, ε_n is the normalised emittance in both planes and I_0 = 31MA for proton. For an isochronous cyclotron the average focusing strength is given by [4], $k_0 = \left(\nu_0 \bar{B}\right)/\left(m\beta\gamma c\right)$. For N sector isochronous cyclotron without spiral and with hill field B_H valley field B_V we have [4]

$$\bar{B} = \frac{N\theta_H}{2\pi}(B_H - B_V) + B_V \qquad (2)$$

$$\nu_z^2 = -\beta^2\gamma^2 + \frac{N^2}{N^2-1}\left[\frac{\left(B_H - \bar{B}\right)\left(\bar{B} - B_V\right)}{\bar{B}^2}\right] \qquad (3)$$

where θ_H is the hill angle. If the beam injected into the cyclotron is non uniform (Gaussian) then the emittance growth is obtained from the equation [3]

$$\frac{\varepsilon_f}{\varepsilon_i} = \left(1 + \frac{0.77K}{a_i^2 k_i^2}\right)^{1/2} \qquad (4)$$

For a given value of beam current we notice that one can minimize the emittance growth by maximizing the focusing strength. For compact cyclotron this can be possible by proper choice of the hill angle and magnetic field flutter.

If the beam injected into the cyclotron is rms mismatched, then it rotates clockwise when propagates along the cyclotron orbit and the effective radius oscillates between a minimum and maximum value [3]. The final beam radius can thus be obtained by solving the following equation,

$$\left(\frac{a_f}{a_i}\right)^2 - 1 - \chi \ln\frac{a_f}{a_i} = h, \qquad \chi = 1 - \frac{k_i^2}{k_0^2} \qquad (5)$$

Where h is a free energy parameter. Finally the emittance growth can be obtained from equation,

$$\frac{\varepsilon_f}{\varepsilon_i} = \frac{a_f}{a_i}\left\{1 + \frac{k_0^2}{k_i^2}\left[\left(\frac{a_f}{a_i}\right)^2 - 1\right]\right\}^{1/2} \qquad (6)$$

where a_f is the final beam radius.

RESULTS AND DISCUSSIONS

Since in a cyclotron the axial focusing force is weak compared to the radial focusing force, emittance growth is more in the axial direction. Here we have considered only the analysis of axial effects, however, the radial effect can be analysed in a similar way. We have used N = 4, fixed value for the hill field B_H = 1.6T and three different values 0.2, 0.4 and 0.6 T for the valley field B_V. The beam parameters are as follows: injection energy = 80 keV, initial normalised emittance ε_n = 0.8πmmmrad and rf phase width $\Delta\phi$ = 30^0. Let us consider the first case in which the injected beam has Gaussian distribution. Using eqns (2-4) we obtained the variation of the axial emittance growth as a function of the hill angle for 5 mA aveage beam current. Results are presented in Fig. 1.

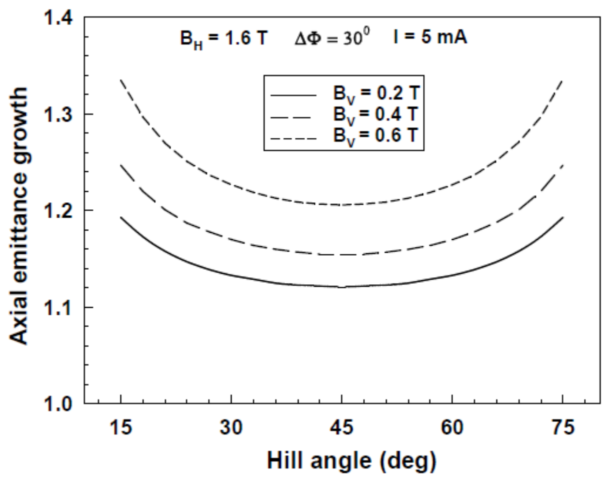

Figure 1: Emittance growth as a function of hill angle.

We notice that the emittance growth reaches minimum for hill angle close to π/4 because in this case the vertical focusing frequency is maximum. We also notice that at a given hill angle the emittance growth increases if (B_H - B_V) decreased. Fig. 2 shows the variation of the emittance growth as a function of the beam current *I* for a constant hill angle π/4.

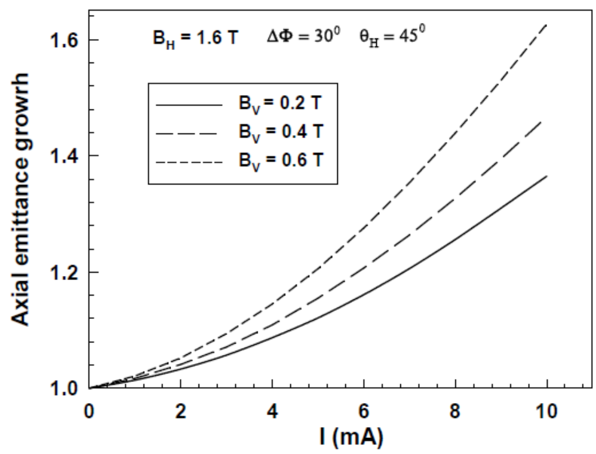

Figure 2: Emittance growth as a function of beam current.

We see that emittance growth increases with the beam current and also for a given beam current it is large for small (B_H - B_V). Fig. 3 shows the variation of the emittance growth as a function of the injection energy for three different values of the beam current. As expected, the emittance growth is large for low injection energy and it decreases as the injection energy is increased. We have observed that, for a given beam current and injection energy, the emittance growth reduces when the flutter is increased and hill and valley angles are kept equal.

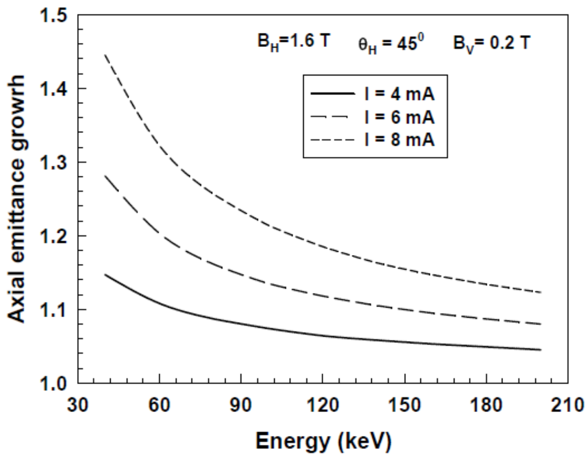

Figure 3: Emittance growth as a function of beam energy.

Let us consider the second case in which the injected beam (current 5 mA) has uniform distribution but the beam size differs from the matched radius. The matched beam radius corresponding to B_H=1.6T, B_V=.2T θ_H=π/4, can be obtained by solving Eqn. (1) and for the present case a_i = 2.688mm. We have varied the injected beam size from 1.5 mm to 4 mm and studied the corresponding emittance growth. Results are shown in Fig. 4. We observe substantial emittance growth for beam radii away from the matched beam radius and effect is more serious for beam radii smaller than the matched beam radius.

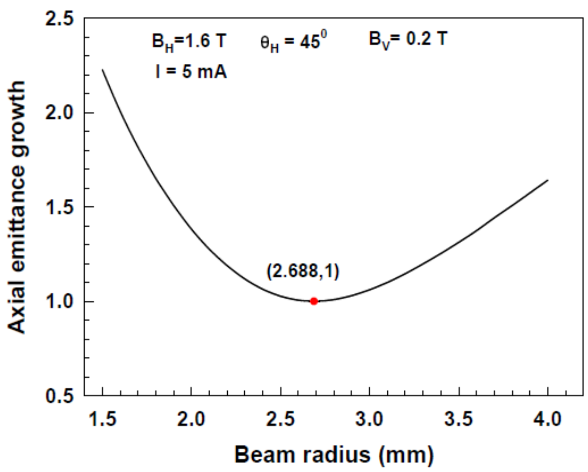

Figure 4: Axial emittance growth as a function of the injected beam radius. Beam current is 5 mA.

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