TRANSPORT OF SPACE CHARGE DOMINATED BEAM THROUGH THE SPIRAL INFLECTOR

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Abstract

We have studied the behaviour of paraxial ion trajectory through the spiral inflector in the presence of linear space charge effect for a 10 MeV compact cyclotron. The initial conditions of the beam have been optimized to reduce the coupling effects in two transverse phase planes at the inflector exit.

INTRODUCTION

At VECC, we are developing a 10MeV, 5mA compact proton cyclotron [1]. A 2.45 GHz microwave ion source presently under testing for beam characterization, will produce ~20mA of proton beam at 80 keV. The extracted beam will be injected axially in the central region of the cyclotron by a tilted spiral inflector [2]. The parameters of the spiral inflector are A = 8.65 cm, R_m = 7.92 cm and k' = 0.65. The electric field E_0 is constant (18.5kV/cm) and the magnetic field B_0 is 5.15 kG.

THEORETICAL ANALYSIS

The effect of space charge in paraxial ion trajectory calculations have been included considering a straight uniform cylindrical beam of constant radius a [3]. The self-induced electric and magnetic fields generated by the space charge are given by:

$$E_r = \frac{I \cdot r}{2\pi\varepsilon_0 a^2 v_0}, \quad B_\varphi = \frac{v_0 E_r}{c^2} \qquad r \le a \tag{1}$$

where I is the beam current, v_0 is the beam velocity. In the first order approximation and neglecting the fringe field effect at the inflector edge, the coupled differential equations for paraxial ion trajectory through the spiral inflector can be written as,

$$\alpha'' = -2\gamma - (2K + k')\beta' \cos b + \alpha + 2\alpha K k' \cos^2 b + (\alpha - \beta k' \sin b)\frac{1 + 2Kk' \sin^2 b}{1 + k'^2 \sin^2 b} + \frac{I A \alpha}{2\pi\varepsilon_0 a^2 v_0 E_0},$$

$$\beta'' = -(2K + k')(\alpha \sin b - \alpha' \cos b - \gamma' \sin b) + 2\beta K k' - (\alpha - \beta k' \sin b)\frac{k' \sin b(1 + 2Kk' \sin^2 b)}{1 + k'^2 \sin^2 b} + \frac{I A \beta}{2\pi\varepsilon_0 a^2 v_0 E_0},$$

$$\gamma'' = -(2K + k')(\beta' \sin b + \beta \cos b) + 2\alpha' \tag{2}$$

$$K = k' + A / 2R_m \qquad b = v_0 t / A$$

where $\alpha = u/A$, $\beta = h/A$ and $\gamma = v/A$. Here u, h and v denote the coordinates of the paraxial ray in the optical coordinate system. Equations (2) are similar to the equations given in reference [3] except for the last terms which are due to the space charge effect. These terms are not correct in reference [3] due to an extra q in the numerator and a missing E_0 in the denominator and hence give incorrect results. Since the space charge term missed a factor $1/qE_0$ which is of the order of 10^{12}, the effect of space charge is almost negligible even for several hundreds of mA of beam current.

The beam divergence p_u, p_h and p_v in the coordinate system normal to the central trajectory velocity vector are,

$$p_u = \alpha' + 2\beta K \cos b \tag{3a}$$

$$p_h = \beta' - 2\alpha K \cos b \tag{3b}$$

$$p_v = \gamma' - \alpha + 2\beta K \sin b \tag{3c}$$

RESULTS AND DISCUSSIONS

We have written a computer code to solve differential equations (2) to obtain the paraxial ion trajectories. We have used the initial emittance equal to 60.0 πmmmrad in both planes. The behaviour of paraxial ion trajectories through the spiral inflector in u and h planes for various values of average beam current are shown in Fig. 1. The effect of space charge is clearly visible. The input conditions are same for all the values of beam current (u=h=3 mm and P_u=P_h=12.5 mrad).

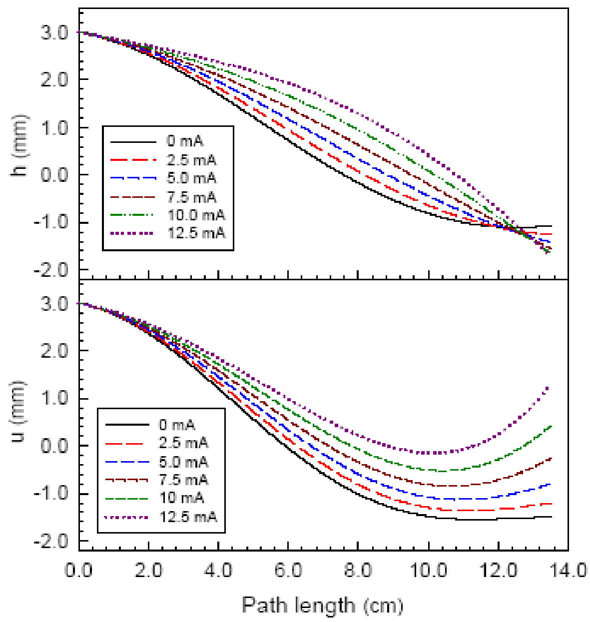

Figure 1: The paraxial ion trajectories in u and h planes.

The paraxial rays of forty representative particles that belong to the boundary of the contours in (u-p_u) and (h-p_h) planes and correspond to ellipses of 60 πmmmrad at the entrance in both the planes were run through the inflector. Initially the orientation of the entrance ellipses with zero current was optimized to have minimum growth in emittance at the inflector exit. We then studied the behaviour of the emittance growth at the exit of the spiral inflector with beam current keeping the initial conditions same. Results are presented in Fig. 2.

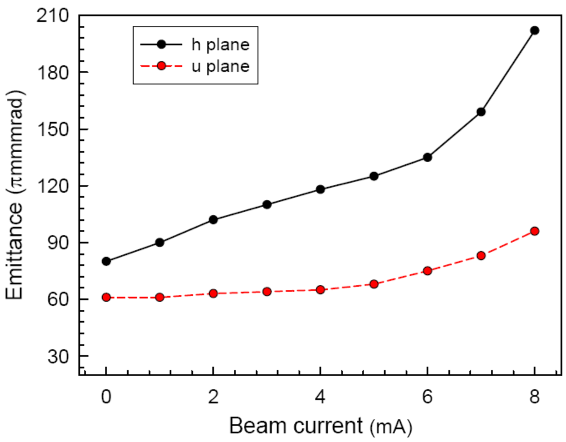

Figure 2: Variation of beam emittance at the exit of the inflector in *u* and *h* planes as a function of beam current.

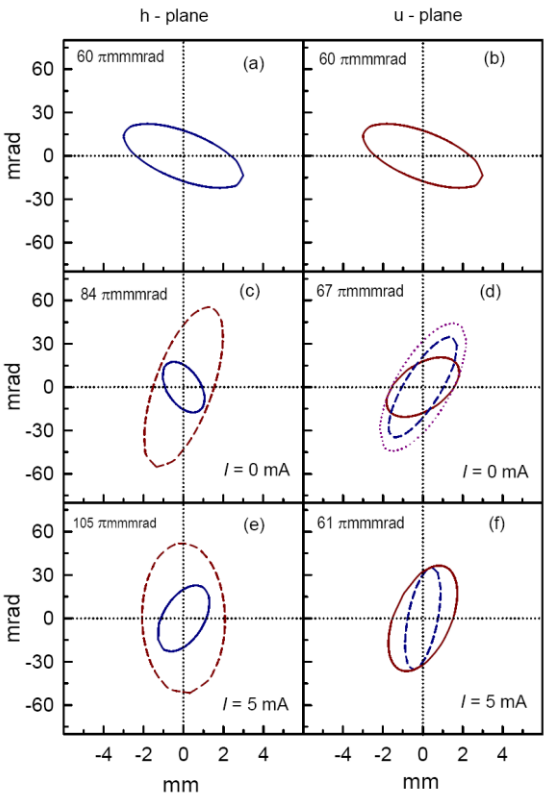

Figure 3: Phase space plot in both planes. a) & b) for entrance; c) & d) at exit for I = 0 mA; e) & f) at exit for I = 5 mA for optimize ellipse. (I_{peak}=50mA)

The effect of inter plane coupling as well as the space charge on emittance growth is more in the *h* plane compared to that in the *u* plane. The initial phase ellipses which give the minimum emittance growth without space charge are not suitable when the space charge effect is included and hence the initial ellipses need to be optimized again to reduce the emittance growth at the inflector exit. Fig. 3 shows the optimized phase ellipses, in *h*-plane and *u*-plane at the inflector entrance and at its exit for I = 0 mA and I = 5 mA. The dilution of emittance in the *h*-plane at the exit of inflector is arising mainly due to coupling effect from *u*-plane. The estimated effective emittances at the exit are shown in the Fig. 3.

The paraxial ion trajectories of forty representative particles around the central ion trajectory and the resulting beam envelope in *u* and *h* planes are shown in Fig. 4 for I = 0 mA and I = 5 mA. The maximum beam size in the inflector is ~ 5 mm which is much less than the minimum gap of 12 mm between the electrodes. It is easy to notice that emittance in *u* plane is major contributor in coupling effect and emittance dilution. Results show that small value of emittance in *u* plane at the entrance of the inflector will be advantageous.

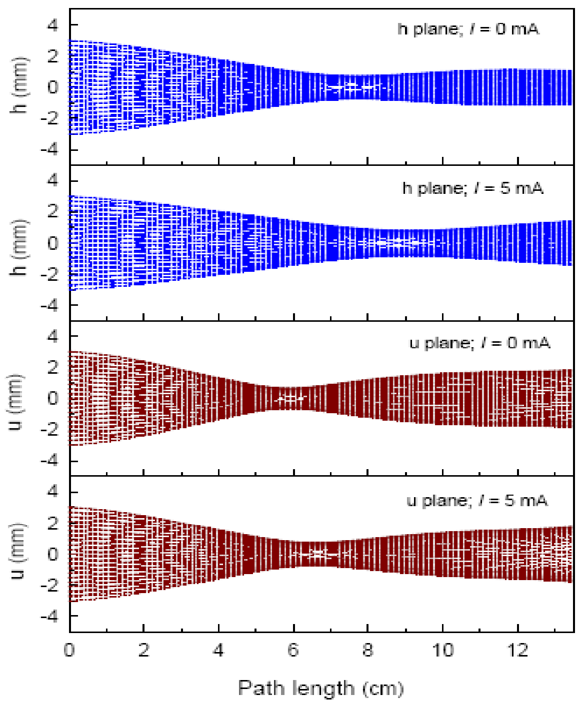

Figure 4: Beam envelopes in the spiral inflector in both *u* and *h* planes for I = 0 mA and I = 5 mA.

REFERENCES

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